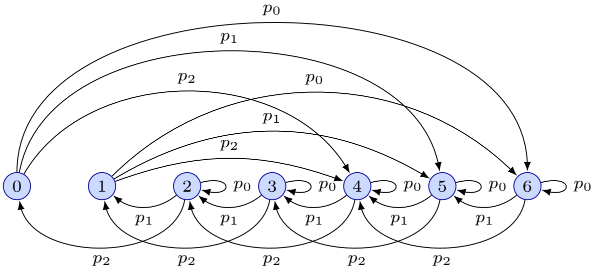
EE365: Hitting Times

Example: Inventory re-ordering

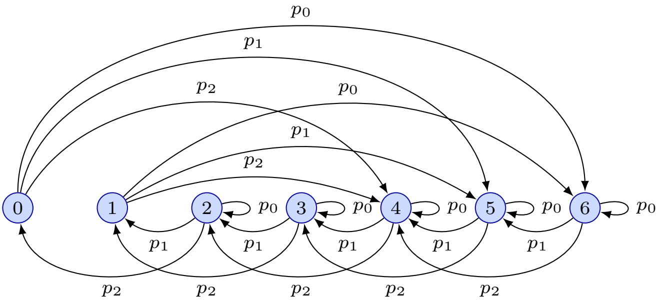
if we start in state C, how long before we re-order?

$$\tau_E(x_0, x_1, \ldots) = \min\{t > 0 \mid x_t \in E\}$$

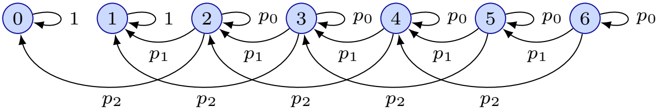
- τ_E is a random variable, called the *first passage time* or *hitting time* to set E
- τ_E is the earliest time when $x_t \in E$
- we set $E = \{0, 1\}$



Computing the distribution of first passage times



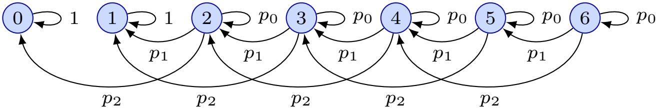
replace states in E (in this case 0 and 1) with *absorbing states*



hittings times to set E are the same for both chains

Computing the distribution of first passage times

let Q be the transition matrix of the new chain



for $j \in E$

$$\mathbf{Prob}\big(\tau_{\{j\}}(x) = t \,|\, x_0 = i\big) = (Q^t)_{ij} - (Q^{t-1})_{ij}$$

i.e., conditioned on $x_0 = i$,

$$\mathbf{Prob}(t \text{ is the first time at which } j \text{ is reached}) =$$
$$\mathbf{Prob}(j \text{ has been reached by time } t) - \mathbf{Prob}(j \text{ has been reached by time } t-1)$$

Example: Inventory re-ordering

- how long before we re-order, given that we start fully stocked?
- plot shows $\mathbf{Prob}(\tau_{\{0,1\}} = t \mid x_0 = 6)$ vs. t (mean is 13.1)