EE365: Epidemic Example

Monte Carlo simulation

to approximate

$$e = \mathbf{E}\, f(x_0, \dots, x_T) = \sum_{s_0, \dots, s_T \in \mathcal{X}} f(x_0, \dots, x_T) d_{s_0} P_{s_0 s_1} \cdots P_{s_{T-1} s_T}$$

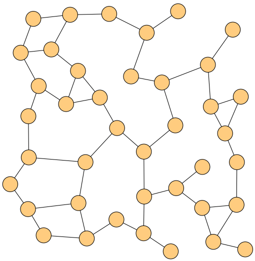
(a sum with n^{T+1} terms)

- simulate N trajectories $x_t^{(i)}$, and let

$$\hat{e} = \frac{1}{N} \sum_{i=1}^{N} f(x_0^{(i)}, \dots, x_T^{(i)})$$

- $\hat{e}$ is an unbiased estimate of e
- works for *any* function f

Example: Epidemic models



- undirected graph
- k vertices, each represents an individual
- each may spread infection to neighbors
- each individual is either susceptible, infected, or removed
- called the S, I, R model
- transition probabilities for each individual depends on infection state of neighbors

- $\mathcal{X} = \{S, I, R\}^k$, so $|\mathcal{X}| = 3^k$
- for modest k, we cannot store a distribution on $\mathcal{X}$
- one approach is to use Monte Carlo methods to estimate useful quantities

Example: Epidemic models

- graph is 30×30 grid
- transition probabilities for an individual, with states ordered (S, I, R)

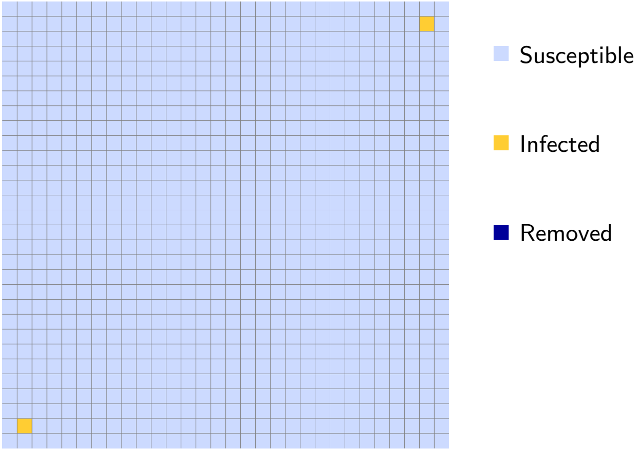
$$\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.6 & 0.2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{with an infected neighbor}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 0.6 & 0.2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{with no infected neighbors}$$

- simulate Markov chain to see spread of infection

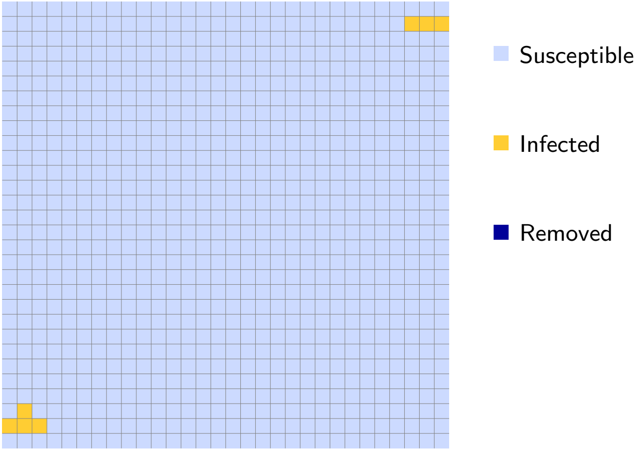
Example: Epidemic models

$t = 0$



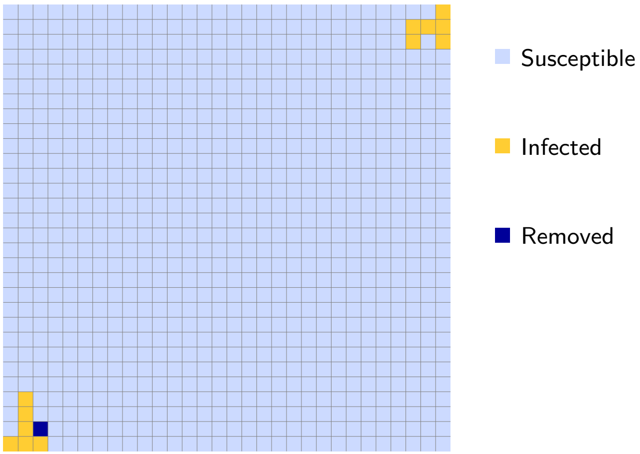
Example: Epidemic models

$$t = 1$$



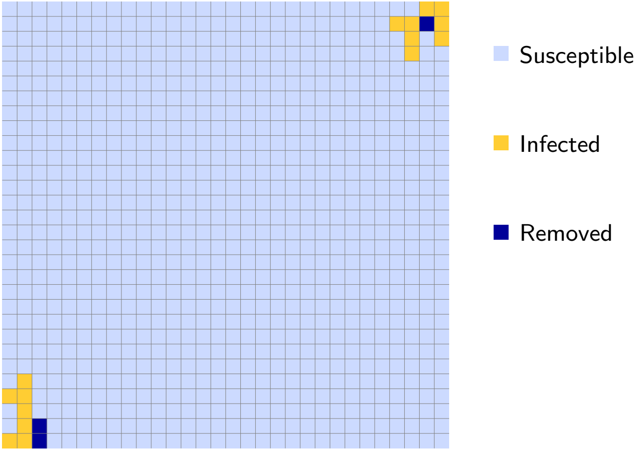
Example: Epidemic models

$t = 2$



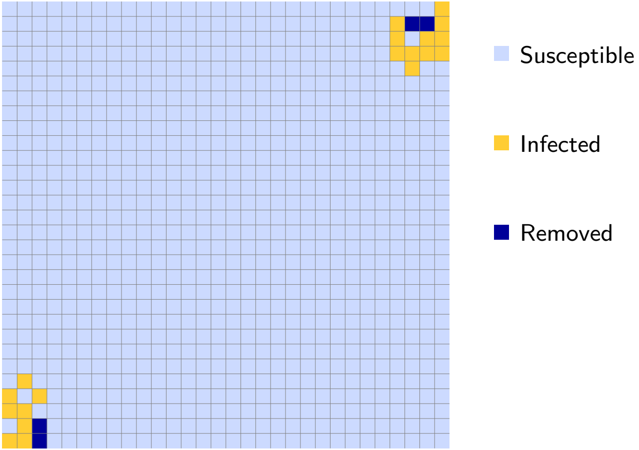
Example: Epidemic models

$t = 3$



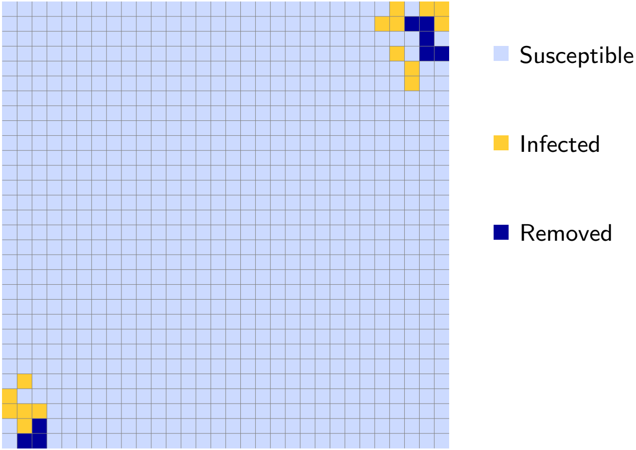
Example: Epidemic models

$t = 4$



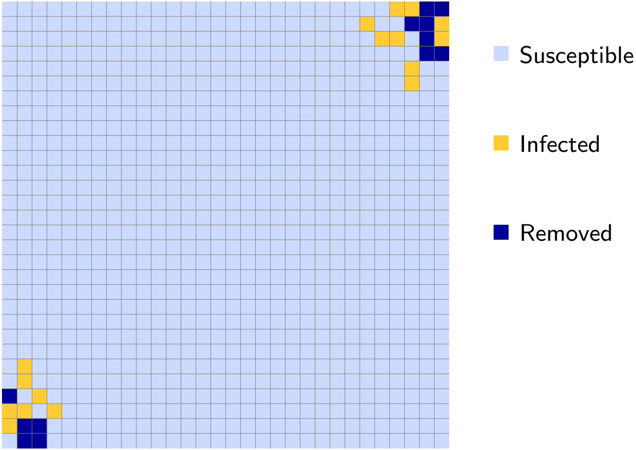
Example: Epidemic models

$t = 5$



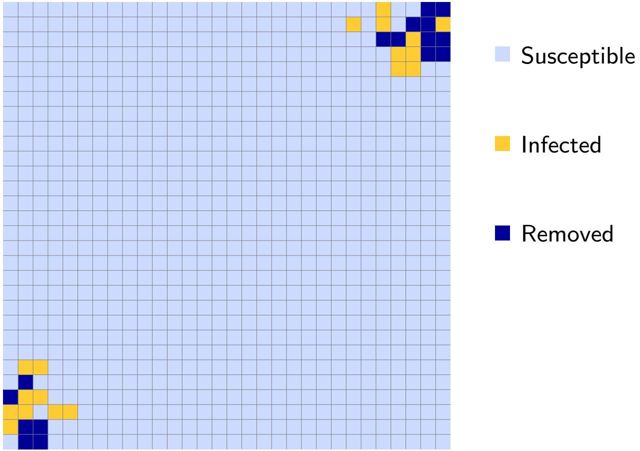
Example: Epidemic models

$$t = 6$$



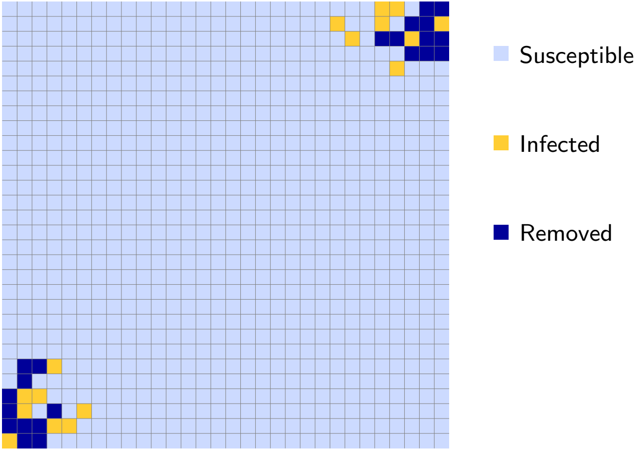
Example: Epidemic models

$t = 7$



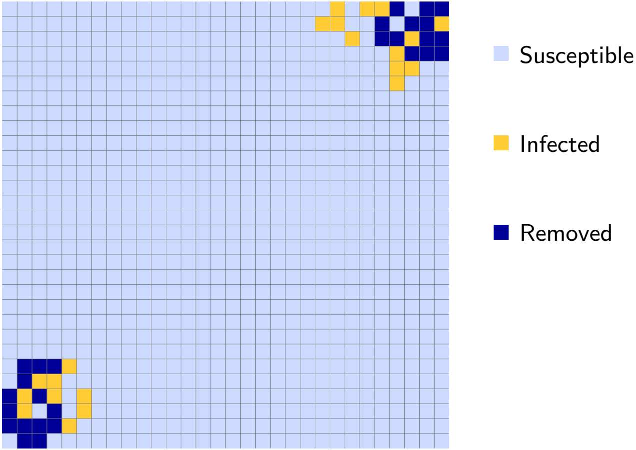
Example: Epidemic models

$t = 8$



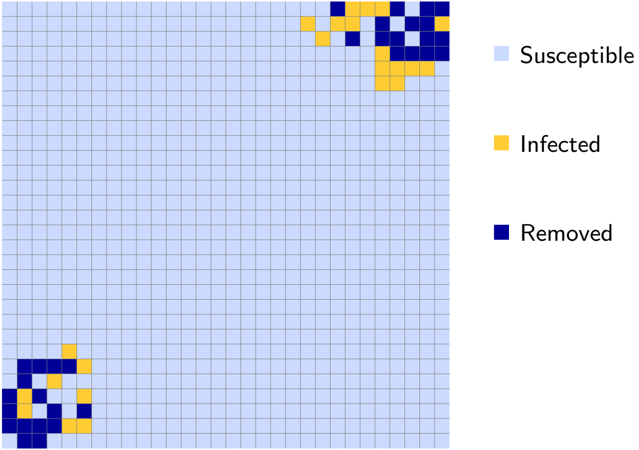
Example: Epidemic models

$t = 9$



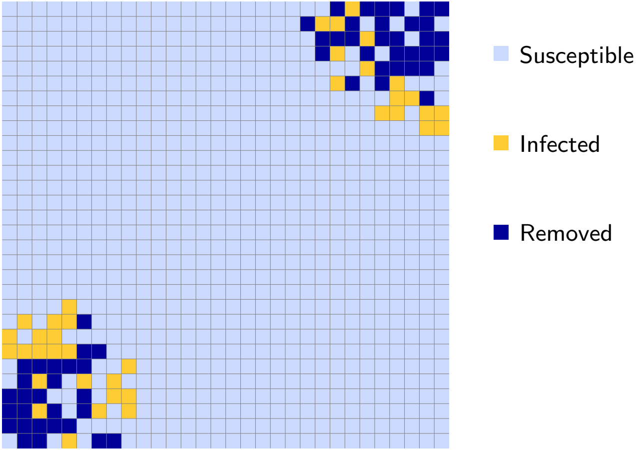
Example: Epidemic models

$t = 10$



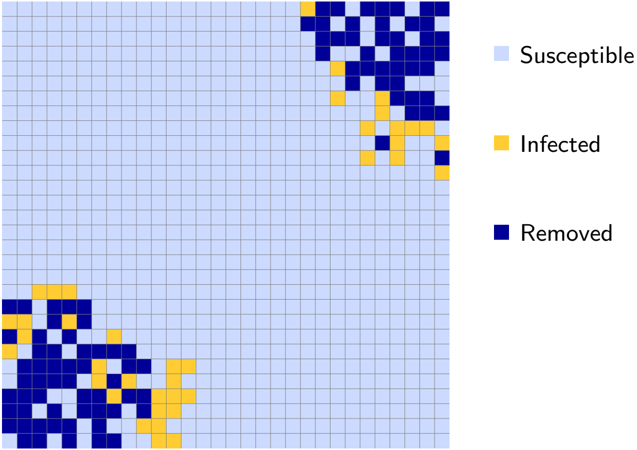
Example: Epidemic models

$t = 15$



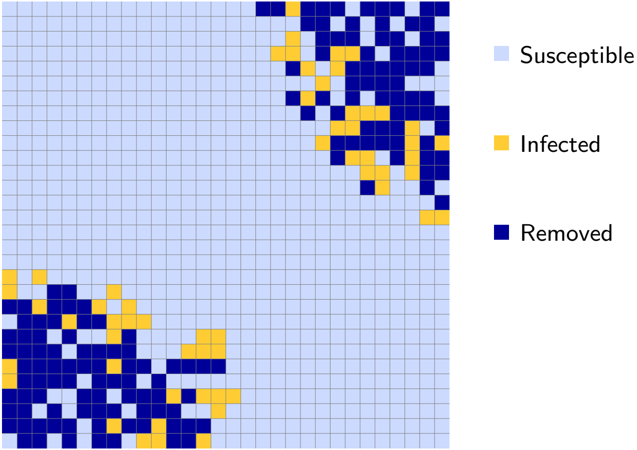
Example: Epidemic models

$t = 20$



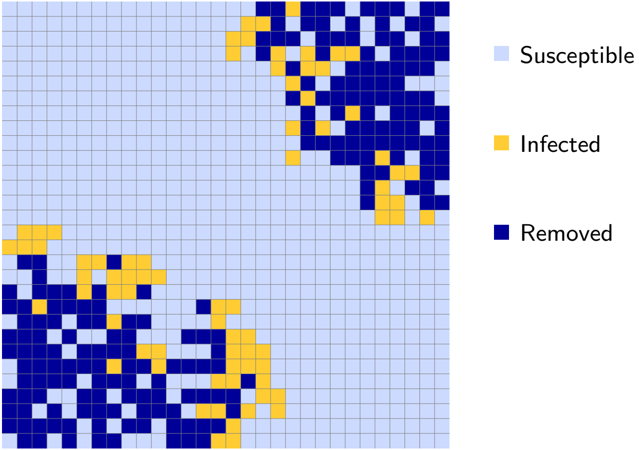
Example: Epidemic models

$t = 25$



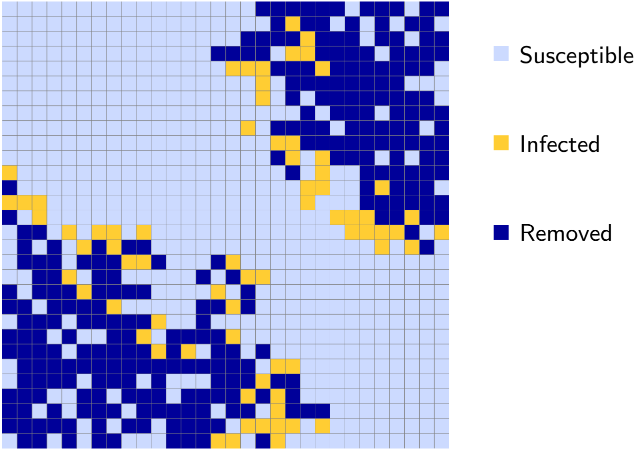
Example: Epidemic models

$t = 30$



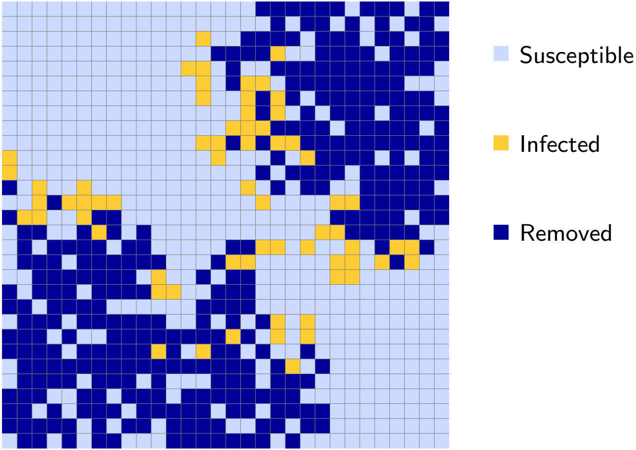
Example: Epidemic models

$$t = 35$$



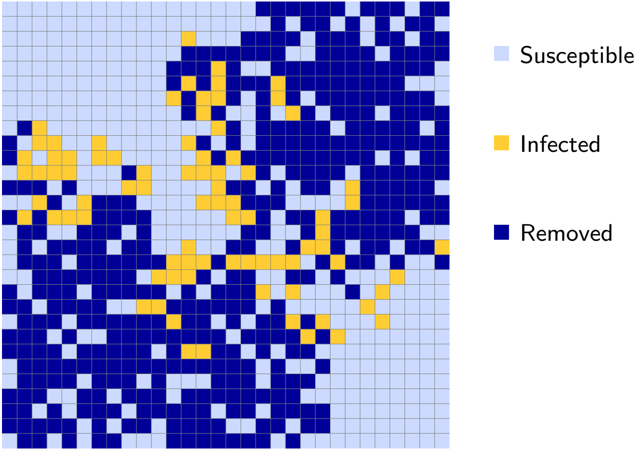
Example: Epidemic models

$t = 40$



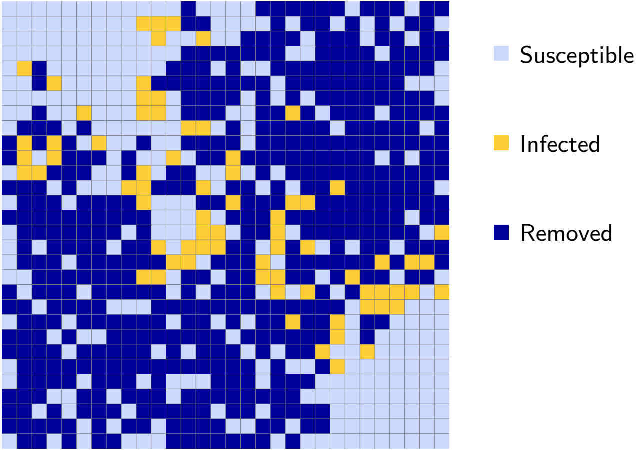
Example: Epidemic models

$t = 45$



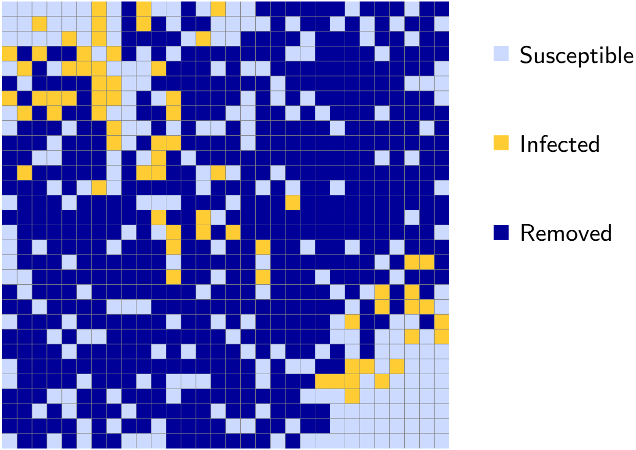
Example: Epidemic models

$$t = 50$$



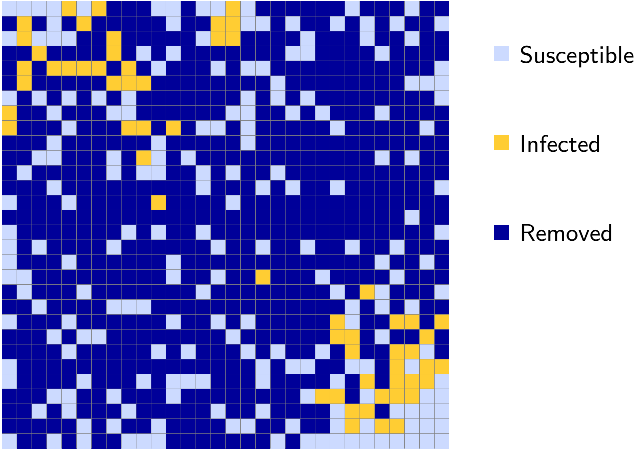
Example: Epidemic models

$t = 55$



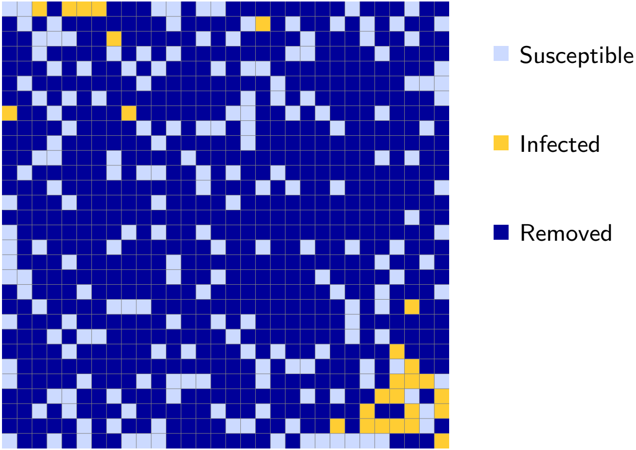
Example: Epidemic models

$t = 60$



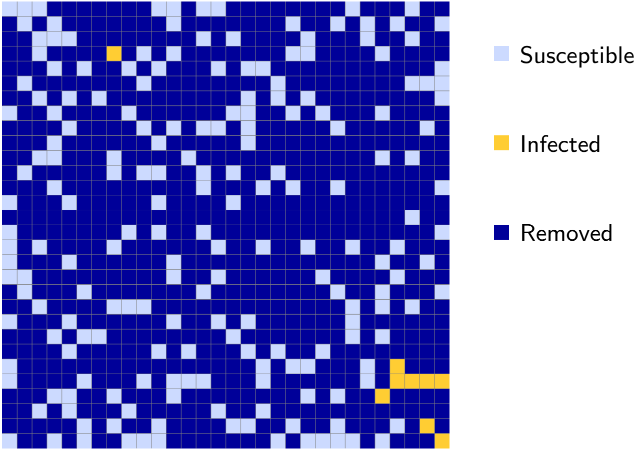
Example: Epidemic models

$t = 65$



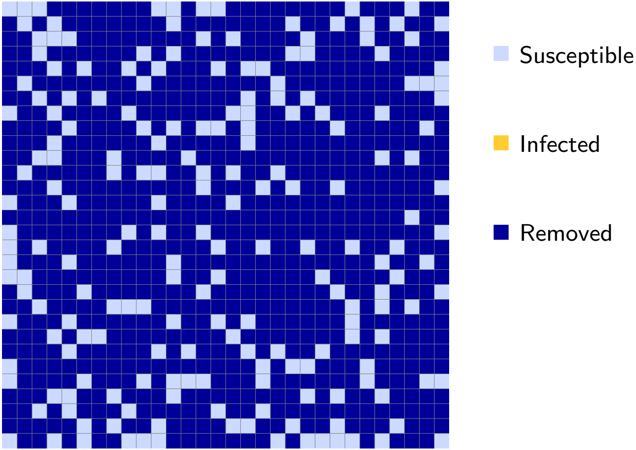
Example: Epidemic models

$t = 70$



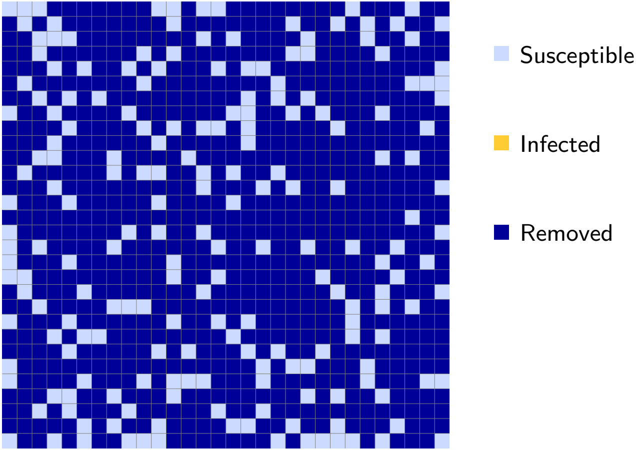
Example: Epidemic models

$t = 75$



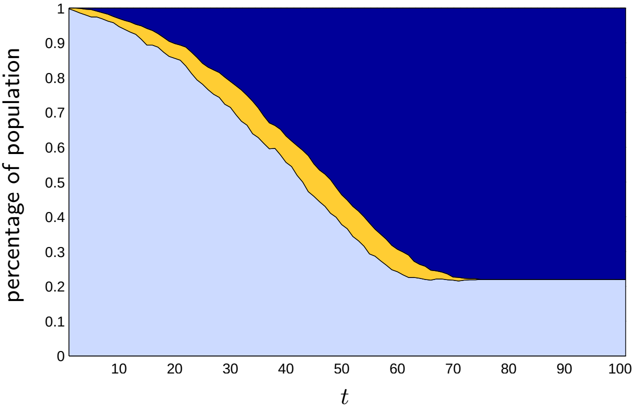
Example: Epidemic models

$t = 80$



Example: Epidemic models

disease spread over time



Example: Epidemic models

- histogram of fraction of population that is removed after 100 time steps
- in 4% of runs, less than 5% is removed